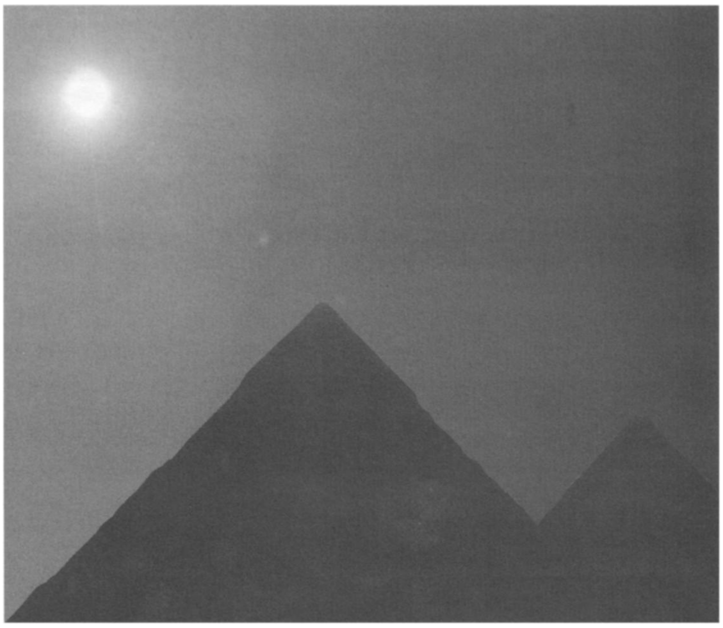
Fractions in Ancient Egyptian Times



by Jack Oliver

This article forms a pair with the article 'Fractions at the Dawn of History: How did the Babylonians Cope with Fractions?' by the same author and gives an outline of some of the contributions that the Egyptians made to the advancement of numeral systems and techniques of calculation.

This article first gives some historical information about the ways in which the Egyptians recorded information. It then describes the hieroglyphics used to record numbers, followed by a description of the way ancient Egyptians conceived of fractions. The paper concludes with some uses of Egyptian fractions between those times and the present.

Introduction

At least as early as 3000 BC, Egyptians carved numbers and hieroglyphics in stone or painted them with the bruised tip of a reed on earthenware or stone or onto sheets of papyrus paper.

Hieroglyphs are pictograms representing people or things. They may be read from right to left when they are oriented to face right (the direction from which they are read) or they may be read from left to right when they are oriented to face left. This is important for our purpose as numbers likewise may be read from right to left or left to right. They could be read top to bottom or bottom to top. The most common way was from right to left followed by from top to bottom. With time, the hieroglyphs evolved to become phonetic representations of consonants (not vowels) as in the case of the Semitic languages (e.g. Hebrew and Arabic).

Alongside the hieroglyphic pictograms there were hieroglyphic numerals. More details of these follow.

Hieroglyphics were difficult to form (write, chisel), so from early times (at least as early as 3000 BC) for everyday purposes, modified more easily formed symbols were used in place of the hieroglyphic numerals. These were the hieratic numerals, which originally were not much different from their hieroglyphic equivalents. Over the centuries, they became more modified and simpler to write.

We might summarize the above by saying that the hieroglyphs were used mainly for monumental purposes and the hieratic script mainly for everyday purposes.

Egyptian Numerals and Numbers

We shall describe Egyptian numerals in terms of the hieroglyphic rather than the hieratic since although the latter are easier to write, the former are easier for us to read.

Like our own number system, the Egyptian number system was strictly decimal. It used the symbols in Figure 1 for the powers of 10.

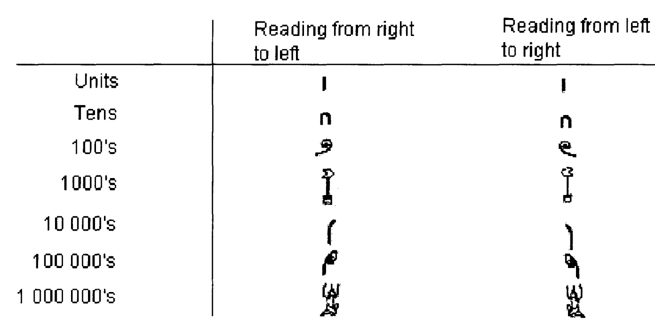


Fig. 1

These were separate from the hieroglyphs representing people and things just as our numerals are separate from our words. There were minor variations of these as there are with our numerals when we write them.

The multiples up to 9 of these powers of 10 were formed by grouping the appropriate number of 10's symbols together. For example 40 was formed by grouping 4 of the 10's symbols together as in Figure 2.

Fig. 2

The multiples of 10 were formed as in Figure 3.

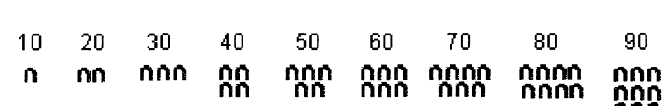


Fig. 3

The other powers of 10 were formed in the same way with their appropriate symbols and with the same geometrical configurations.

Now, how was all this put together to form a positive integer? We shall illustrate on the number that we would write as 3456 (Fig. 4).

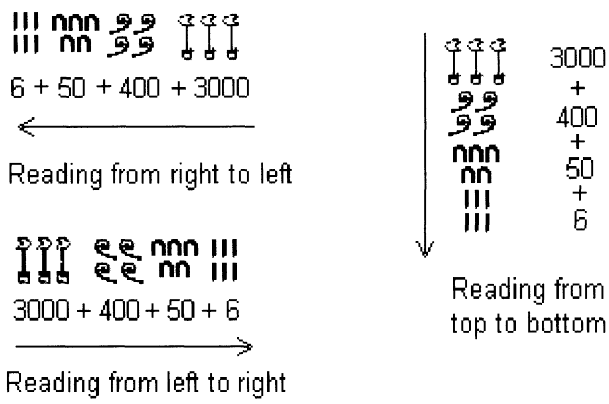


Fig. 4

Egyptian Fractions

The Egyptian idea of fractions was completely different from that of their neighbours in Mesopotamia. For reasons that we have not been able to determine, they only thought in terms of 'Unit Fractions'. To illustrate, they would think of our fraction $\frac{3}{4}$ as $\frac{1}{2} + \frac{1}{4}$ where the numerators of the two terms were both 1's, hence the term 'Unit Fraction'. Also for a reason that we do not know, they would not think of $\frac{2}{5}$ as $\frac{1}{5} + \frac{1}{5}$ but perhaps as $\frac{1}{3} + \frac{1}{15}$ or $\frac{1}{4} + \frac{1}{10} + \frac{1}{20}$ since there is no unique way of resolving a common fraction into a sum of unit fractions. But for more information, see the end note [1].

In hieroglyphics, a unit fraction was formed by placing the hieroglyph

(mouth symbol, R sound) over the hieroglyphs for the value of the denominator. See Figures 5 and 6 below for examples:

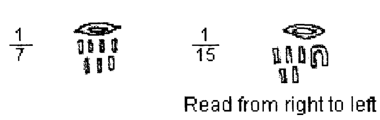


Fig. 5

Or where the denominator was more complicated:

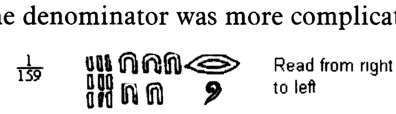


Fig. 6

The very commonly occurring fractions had special signs:

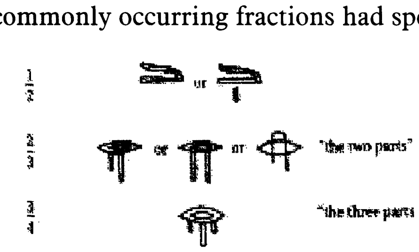


Fig. 7

'Sharing' problems are problems where 'nice' solutions can be obtained by using unit fractions. Perhaps these were the inspiration for the Egyptians.

Problem: Divide 8 loaves equally among 5 people.

Our solution:

$\frac{8}{5} = 1\frac{3}{5}$

And each person gets 1 loaf and $\frac{3}{5}$ of a loaf (How do we cut 5 lots of $\frac{3}{5}$ of a loaf from the original loaves?)

Egyptian solution:

8 divided by 5 corresponds to $1 + \frac{1}{2} + \frac{1}{10}$
and each person gets 1 loaf, $\frac{1}{2}$ a loaf an $\frac{1}{10}$ of a loaf as shown in Figure 8.

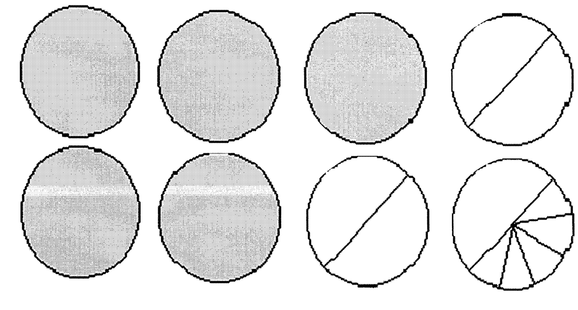


Fig. 8

Problems similar to the above (but considerably more difficult) are to be found in the Rhind Papyrus. Also see end note [2].

Moving down the stream of time, Heron of Alexandria (1st century AD) in the solution to a problem in mensuration, expressed our $4\frac{4}{5}$ as $4\frac{1}{2}\ \frac{1}{5}\ \frac{1}{10}$ although at this time astronomers and other scholars used the sexagesimal fractions of the Babylonians to write down fractions.

Over a thousand years later (AD 1202), Fibonacci was fond of using unit fractions. In *'Liber abaci'* we find:

$\frac{98}{100} = \frac{1}{1000} + \frac{1}{50} + \frac{1}{5} + \frac{1}{4} + \frac{1}{2}$ and $\frac{99}{100} = \frac{1}{25} + \frac{1}{5} + \frac{1}{4} + \frac{1}{2}$

In more recent times, when the author was at school in the 1930's he was taught how to use 'aliquot parts'. This will amuse some of the author's older readers. We have a problem.

What is the cost of 27 articles at £3 .. 13s .. 7d each?
A solution using aliquot parts.

	£	s	d
27 at £3	81	0	0
27 at 10s (= £$\frac{1}{2}$)	13	10	0
27 at 2s 6d (=£$\frac{1}{8}$)	3	7	6
27 at 1s (= £$\frac{1}{20}$)	1	7	0
27 at 1d (= £$\frac{1}{240}$)		2	3
27 at £3 .. 13s .. 7d	£99 ..	6s ..	9d

Today, there might be the occasional problem where the idea of unit fractions could produce an interesting solution, but the expression of fractions in this way seems to be one of the dead ends of mathematics.

End Notes

[1] *Decomposition into 'Unit Fractions'.*

The Rhind Papyrus (see end note [2]), opens with a table of unit fractions expressing $\frac{2}{n}$ as a sum of unit fractions for n

= 5(2)101. The following are some examples (using the author's notation):

$\frac{2}{5} = \frac{1}{3} + \frac{1}{15} = [3,15]$

$\frac{2}{11} = [6,66]$

$\frac{2}{15} = [10,30]$

$\frac{2}{101} = [101,202,303,606]$

One way of producing decompositions of $\frac{2}{n}$ into unit fractions is to use (the equivalent) of

$$\frac{2}{n} = \frac{1}{\frac{n+1}{2}} + \frac{1}{\frac{n(n+1)}{2}}$$

or

$$\frac{2}{pq} = \frac{1}{p\frac{p+q}{2}} + \frac{1}{q\frac{p+q}{2}}$$

When $n = 15$, these give:

$\frac{2}{15} = [8,120]$
$\frac{2}{15} = [12,20]$

Some of the items in the $\frac{2}{n}$ table could be found as above but not all (and not $\frac{2}{15}$)

The table value of $\frac{2}{15}$ could be found from the equivalent of:

$$\frac{2}{3} \cdot \frac{1}{p} = \frac{1}{2p} + \frac{1}{6p}$$

by putting $p = 5$ when

$\frac{2}{15} = [10,30]$

The unit fractions of $\frac{2}{5}$ could be found as follows:

Take $\frac{1}{3}$ ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$ had pride of place in the Egyptian system) of $\frac{1}{5}$ (half of $\frac{2}{5}$) and then try to find a unit fraction to add to this to produce $\frac{2}{5}$. In our notation

$\frac{2}{5} = \frac{1}{15} + r$ when $r = \frac{1}{3}$ and $\frac{2}{5} = [3,15]$

If we had taken $\frac{1}{2}$ of $\frac{1}{5}$ we get

$\frac{2}{5} = \frac{1}{10} + \frac{1}{5} + \frac{1}{10}$

which was not acceptable to the Egyptians.

There is some indication in the Rhind Papyrus that this method was used with some fractions.

A systematic method used by Fibonacci in (*'Liber abaci'*, AD 1202) on any fraction proceeded as follows on the example $\frac{13}{21}$:

Write $\frac{13}{21} = \frac{1}{2} + r_1$

where $\frac{1}{2}$ is the largest unit fraction such that $\frac{1}{2} < \frac{13}{21}$ when $r_1 = \frac{5}{42}$

Write $\frac{5}{42} = \frac{1}{9} + r_2$ ($\frac{1}{9}$ being the largest unit fraction such that $\frac{1}{9} < \frac{5}{42}$

When $r_2 = \frac{1}{126}$ and $\frac{13}{21} = [2,9,126]$ It can be shown that this process is always finite.

[2] *The Rhind Papyrus*

The Rhind Papyrus dates back to about 1650 BC and along with the Moscow Papyrus (about 1850 BC), they contain much of what we know about ancient Egyptian mathematics.

The Egyptian scribe Ahmes, seems to have copied the papyrus from earlier documents (now lost) which went back a further 200 or more years before his time. The papyrus itself is about 30 cm high and 540 cm long. It had been bought in Luxor, Egypt in 1858 by a Scottish antiquary, Henry Rhind. It is now in the British Museum with some fragments in the Brooklyn Museum.

The contents of the papyrus are a collection of 85 problems being mathematical exercises and practical examples on fractions, simple equations, progressions and mensuration of areas and volumes.

References and Further Reading

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