Masters' Writings and Students' Writings: School Material in Mesopotamia

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Abstract: By nature, school drafts of Mesopotamia were meant to destruction. But, thanks to clay support of writing and ancient recycling practices, they reached us in vast amount. These school tablets were mainly produced along a quite short period, between 18th and 17th century B.C. They contain principally exercises for learning writing, Sumerian language and mathematics. These sources bear witness of phenomena linked with those which are examined in this book: change of knowledge medium, from memorisation to writing; standardisation processes, notably in the field of writing and metrology, in which scribe schools played a key part; emergence of a set of ideological references specifically linked to a scholar milieu. This contribution relies on school sources, particularly mathematical ones, in order to bring to light some aspects of these phenomena. In particular, it endeavours to identify authors, users, function and status of different types of mathematical writings produced in the context of scribal schools.

Key words: Ancient Near East; Cuneiform writing; Curriculum; Mesopotamia; Metrology; Nippur; Old Babylonian period; Scribal school; Sexagesimal place value notation; Standardization

9.1 Introduction

This chapter offers a reflection on masters’ documentation in the context of the scribal schools that flourished in Mesopotamia about four thousand years ago. The approach is historical but, despite the fact that the Mesopotamian sources are very distant from us, the issues addressed here are similar to some of the phenomena analyzed by other authors of this book.

Let us evoke some of these phenomena. The development of scribal schools in the late third millennium and the early second millennium in Mesopotamia corresponds to a switch in the medium used for the accumulation and transmission of knowledge: from memorization, the medium became essentially written during this period. This switch is in a way symmetrical to the ones described in this section (Ruthven, Chap. 5). The normative function of the curriculum (Remillard, Chap. 6) is a striking feature of the ancient system of education, and this function explains the fact that that the written artifacts which reached us are highly stereotyped (section 9.5). Literary texts found in ancient schools give evidence of the emergence of an ideology which legitimates the schools and the stratum of erudite scribes that the schools produce (section 9.4). The impact of historical and cultural context on the masters' activities is emphasized in section 9.4 of this chapter as well as in Chap. 16 (Gueudet & Trouche). The collective aspect of creation and transmission of knowledge is addressed in the fourth part of this book; this dimension appears likewise in section 9.6.

In the following, I intend to show how Mesopotamian sources shed light on the documentation work of the most ancient teachers we know.

9.2 Documentation in scribal schools.

The word "documentation" conveys different meanings according to the communities of scholars who use it. For historians, documentation is generally a set of written artifacts that provide information about a given problem. In the present book, documentation is rather understood as a process, by which teachers transform available resources into documents for teaching (Gueudet & Trouche, Chap. 16). In order to avoid ambiguities, I shall use the word "sources" to designate the written artifacts used by historians, and the word "resources" to designate the knowledge, memorized or written, used by the masters of the scribal schools.

What kind of resources were the ancient masters using? In order to answer this question, historians cannot employ the same investigation methods as observers of present-day realities. Historians are dependent on their sources, that is, on a small body of evidence which illuminates only a limited part of ancient *realia.* Moreover, the picture they paint could be distorted by the work of interpretation that they have to do in order to make the data intelligible. However, concerning Mesopotamian scribal schools, the situation is exceptionally favorable due to the huge quantity of school tablets handed down to us (section 9.3).

No other educational system of the past is as well documented as that of Mesopotamia. But the sources that we have provide a truncated view of life in the schools, because it is mainly the production of students that has been preserved[1]. Indeed, if the curriculum of elementary education is quite well known, the masters' activities, their relationship with students or peers, their status, their

[1] The historian of education very rarely has the chance to have access to students' work (drafts, notebooks, exams), which had no value in the eyes of its authors, and was generally destroyed.

personality, their sources of income, and their role in the city, are few documented. We know even less about the material available to them, their libraries, if any.

By combining information from various sources, we can nevertheless shed some light on the work of the masters. Direct information is provided by samples of Sumerian literary compositions written by masters for students (section 9.4). Indirect information on masters is provided by the writings of their students (section 9.5). More evidence comes from texts written by masters for other purposes than elementary education (section 9.6).

9.3 Sources and historical context

The sources which provide information on education in Mesopotamia are mainly school tablets, that is, clay tablets written by young students during the first stage of their education (or “elementary level”)[2]. These tablets were discovered in many archaeological sites, over a large geographical area, including present-day Iraq, Iran, Syria and Lebanon. On these tablets, young scribes wrote out exercises for learning cuneiform writing, Sumerian vocabulary and grammar, numbers, measures and calculations. The future careers of the students were probably quite diversified. Many of them were prepared for administrative tasks in various levels of the administration of temples, palaces, or private household. Some of them were probably more specifically prepared to ensure the transmission of knowledge. It is difficult to describe with more precision either the future professions of the scribes educated in schools or the scholarly crafts. What is clear is that the schools, particularly those of Nippur (see below), played a key role in training social elites. It is also probable that the dynamic extension of scribal schools in the late third millennium (Neo-Sumerian period) and the early second (Old-Babylonian period) in Mesopotamia and beyond was accompanied by the development of a scholarly milieu linked to these schools. It seems that, at least in important schools such as those of Nippur, education was provided by professional scholars, who were quite specialized (see text “Edubba D” below). These scholars were perhaps at the same time instructors in charge of the young beginners, professors in charge of the advanced students, and creative scientists. They could occupy at the same time high

[2] Veldhuis (1997). We don’t know how old the students were at the beginning of their scribal education. They were old enough to be able to manipulate clay and “calame” (the cane the scribes used to impress signs on wet clay), but still in the charge of their parents. Moreover, the age of the students could have changed according to the place and the period.

positions in the temple, the city or the palace hierarchy. Since we ignore the exact nature of the scholars' charge, I prefer to refer to them as "masters" rather than as "teachers", a term which could implicitly suggest that teaching at the elementary level was their unique activity.

The conservation of the unskilled writings of students is partially accidental. It is due primarily to the nature of the writing medium, the clay, a nearly indestructible material. It also ensues from the reuse of dry and waste tablets as construction material. Trapped in walls, floors or foundations of houses, tablets produced by students and subsequently discarded have escaped other forms of destruction.

The context in which the education of the scribes took place is not always well known, and it was probably not the same everywhere, or at every time. On the basis of the sources on which they rely, historians point out that the context of the training was institutional, domestic, professional or religious. In Ur, a city of southern Mesopotamia, the home of a priest seems to have housed important teaching activities in the Old Babylonian period (Charpin 2008). In Sippar, farther north, a school was integrated into a household, according to Tanret (2002). The Assyrian merchants who developed a business over long distances between Mesopotamia and Anatolia transmitted the basics of writing and arithmetic to their children by practicing their craft, using methods similar to apprenticeship (Michel 2008). In Nippur, the great religious and cultural capital of Mesopotamia, situated a hundred kilometers south of the present-day Baghdad, the school context appears to have been institutional and secular. Education was especially important because of the presence of a high court, the development of an important medical tradition, and the political role of scribes from Nippur in the legitimization of kingship. If we consider the situation a thousand years later in Mesopotamia, the context has changed completely. The practice of cuneiform has declined in favor of alphabetic writing on perishable media (parchment and papyrus, nowadays lost), and has become confined to restricted circles of scholars, related to the clergy.

This brief historical overview stresses the diversity of the teaching contexts and of the sources used by historians. Each of these contexts deserves particular analysis, which is not possible in the limited space of this chapter. Concerning elementary education (section 9.4 & 9.5), I shall limit myself to the Old Babylonian sources from Nippur, on one hand because of the abundance of tablets, on the other hand because the education in Nippur was a model throughout the vast area in which cuneiform writing was used[3]. The section on masters' writing (section 9.6) will rely on sources from various known or unknown provenances.

9.4 Literary sources

Literary sources which contain information on scribal schools ("Edubba" in Sumerian) are compositions that were used for learning Sumerian vocabulary and grammar. In Nippur the students were taught in Sumerian, a dead language at that time. Akkadian, a Semitic language, had been widespread in Mesopotamia during the third millennium to the detriment of the Sumerian, which had probably disappeared as a mother tongue in the early second millennium. Sumerian, however, remained a scholarly and liturgical language until the disappearance of the practice of cuneiform writing in the beginning of our era.

Had the literary texts used in schools been written specifically for teaching? It is likely that school masters created original compositions, but they also reused old material, transmitted by written or oral tradition. Some hymns praising the skills of Neo-

[3] Old Babylonian tablets from Nippur and now kept in Istanbul and Jena are published in (Proust 2007, Proust 2008a). Photos and informations are available on line at http://cdli.ucla.edu/ (Cuneiform Digital Library Initiative website), by entering Museum number or CDLI number (both information are provided here). Parts of the Philadelphia tablets are published in (Robson 2001). Veldhuis' Ph.D thesis contains a study of lexical tablets from Nippur, and a detailed reconstruction of Nippur curriculum (Veldhuis 1997).

Sumerian kings and their Old-Babylonian successors may have been extracted from a hagiographic literature developed outside of the schools. Other literary compositions appear to be mere products of the schools. Vanstiphout (1978, 1979), for example, has shown that the first literary text studied in the school curriculum is a hymn praising the king of Isin Lipit-Estar (1934-1924). It presents all the characteristics of a text specifically built for the teaching of Sumerian grammar. By its organization, this text allows the systematic learning of a vast repertoire of cuneiform signs, grammatical constructions and rhetorical patterns. Another interesting aspect of this text is the information it gives on the links between schools and society. We learn, for example, that the scribes are supposed to acquire skills in the fields of accounting, law and surveying. This text, as well as others of the same kind, reveals the ideological role of this education, and shows that the students, or at least a part of them, were intended to belong to a caste devoted to the king (Michalowski 1987, p. 63).

Compositions named "Edubba texts" by Sumerologists evoke more directly everyday life in schools[4]. The following picture of a school, extracted from the composition "Edubba A" or "School days" (Kramer 1947, p. 205), gives the impression that the school was an institution where the discipline was harsh, and the staff very numerous and specialized.

Who was in charge of […] (said) "Why when I was not here did you talk?" caned me.

Who was in charge of the […] (said) "Why when I was not here did you not keep your head high?" caned me.

Who was in charge of drawing (said) "Why when I was not here did you stand up?" caned me.

Who was in charge of the gate (said) "Why when I was not here did you go out?" caned me.

Who was in charge of the […] (said) "Why when I was not here did you take the […]?" caned me.

Who was in charge of the Sumerian (said) "You spoke […]" caned me.

My teacher (said) "Your hand is not good," caned me.

[4] We know six *Edubba* texts. The Electronic Text Corpus of Sumerian Literature (ETCSL, http://etcsl.orinst.ox.ac.uk/) provides the following list: Edubba A or "Schooldays" (Kramer 1949); Edubba B or "A scribe and his perverse son" (Sjöberg 1973); Edubba C or "The advice of a supervisor to a younger scribe" (Vanstiphout 1996, Vanstiphout 1997); Edubba D or "Scribal activities" (Civil 1985) – see below; Edubba E or "Instructions of the ummia"; Edubba R or "Regulations of the Edubba". A French translation of "Edubba A" by Pascal Attinger, with philological notes, can be found at: http://www.arch.unibe.ch/content/e8254/e8548/e8549/index_ger.html?preview=preview&lang=ger&manage_lang=ger.

Another Edubba composition provides more details on the curriculum. The text “Eduba D” (Civil 1985) contains a dialogue between two students who are training to speak Sumerian. In turn, they praise their own skills and insult their partner. The text begins as follows (translation Vanstiphout 1997, p. 592)[5]:

(*Examiner and Student*)

1. “Young man, [are you a student?” - “Yes, I am a student.”]

(*Examiner*)

2. “If you are a student,

3. do you know Sumerian”

(*Student*)

4. “Yes, I can speak Sumerian.”

(*Examiner*)

5. “You are so young; how is it you can speak so well?”

This extract shows that the Sumerian was not the mother tongue of the young scribes, who had to learn to speak fluently this dead language. The following extracts refer to the curriculum (*ibid*).

11. The [texts] in Sumerian and Akkadian, from A-A ME-ME

12. [To…] I can read and write.

13. All lines from ^{d}INANA-TEŠ$_2$

14. Till the « beings of the plain » at the end of LU$_2$-*šu* I wrote.

15. I can show you my signs,

16. Their writing and their interpretation; and this is how I pronounce them.”

[…]

19. “Even if I am assigned LU$_2$ = šu on an exercise tablet

20. I can give the 600 LU$_2$ entries in their correct sequence.

[…]

26. In a single day, the teacher would give me the same pensum four times.

27. In the final reckoning, what I know of the scribal art will not be taken away!

28. So now I am master of the meaning of tablets, of mathematics, of budgeting,

29. Of the whole scribal art, of the disposition of lines, of evading omissions, of …

30. My teacher approved (my) beautiful speech.

[5] See also Civil 1985, p. 71-72.

31. The companionship (in the school) was a joyful thing.

32. I know my scribal art perfectly;

33. Nothing flusters me.

34. My teacher had to show me a sign only once,

35. And I could add several from memory."

The text shows the relationship between writing, speaking (line 12 and 30) and memorization (line 35). The list of school tasks enumerated in lines 11, 13, 14, 19, 20, and 28 reproduces the elementary curriculum which was reconstructed from other sources, mainly school exercises (Veldhuis 1997). Note the details concerning some technical skills (lines 19 and 29): shaping the "exercise tablet", drawing the lines between rows and columns, arrange correctly the cuneiform signs, and introducing hyphenation in the right places.

As indicated above, it must be kept in mind that these texts were composed and used for educational purposes, and they deliver an idealized picture of the schools. Several historians have insisted that this kind of literature tells us more about the ideology of the scribes than about the realities of teaching (George 2005). Some details described in "Edubba texts" are nonetheless corroborated by other sources. These compositions are thus sources of information of great value to the historian.

To sum up, the literary texts show that the documentary material used for teaching Sumerian was composite. Much of this material escapes us forever, namely, the whole oral tradition. The written material includes compositions specifically created for teaching, pieces of ancient literary heritage elaborated in earlier periods and recomposed. Once fixed in a curriculum and gradually standardized, this set of texts gained some stability, and passed without much change from one generation to another during the Old Babylonian period and partly beyond. The corpus of literary texts used in education was formed after a complex process mixing original creation, selective reuse of earlier knowledge, and standardization. I will focus further on these processes in the case of mathematics.

9.5 School mathematical tablets

Let us now examine the students' writings. The city of Nippur has yielded several thousands of school tablets, and among them more than 900 contain mathematical texts. For the historian trying to grasp the ancient practices, several aspects should be considered: the content of texts, of course, but also how these texts are inscribed on clay tablets (layout, structure, arrangement), the types of tablets (shape, size), physical condition, and some quantitative data. All these observations allow us to reconstruct a fairly accurate picture of the curriculum, the pedagogical methods, and the concepts taught, particularly concerning numbers and calculations. I shall limit myself to a brief summary of the studies on this teaching context (for more details, see Veldhuis 1997, Robson 2001 and Proust 2007), and stress the information that these sources provide us concerning the resources of the masters, when possible.

From the first glance at the school tablets, one is struck by their material aspect. The school tablets can be classified in four clearly recognizable types. It is interesting to note that certain types of tablets bear names in Sumerian, which means that this classification is not only a convenience of the modern historian, but reproduces the one that the scribes themselves had established. This typology is not quite the same from one site to another, which shows that, beyond the uniformity of the content, teaching methods might vary from one school to another. The types of school tablets from Nippur are the following:

Type I tablets are large tablets containing a long text, continuously and densely inscribed on the obverse and on the reverse (see tablet Ist Ni 2733 on the CDLI website, n° P254643).

Type II tablets contain different texts on the obverse and on the reverse. On the obverse, a model was noted in an archaic style by a master, and copied once or twice by a student; the copies were sometimes traced and erased repeatedly[6]. On the reverse, a dense text was written by heart by a student. Perhaps the Sumerian term "tablet to throw" (saršuba) is associated with this type of tablet (see tablet HS 1703, CDLI n° P229902, containing a lexical list on the obverse and a metrological[7] list on the reverse - Figure 1 below). I shall return later to this type of tablet, particularly important in Nippur.

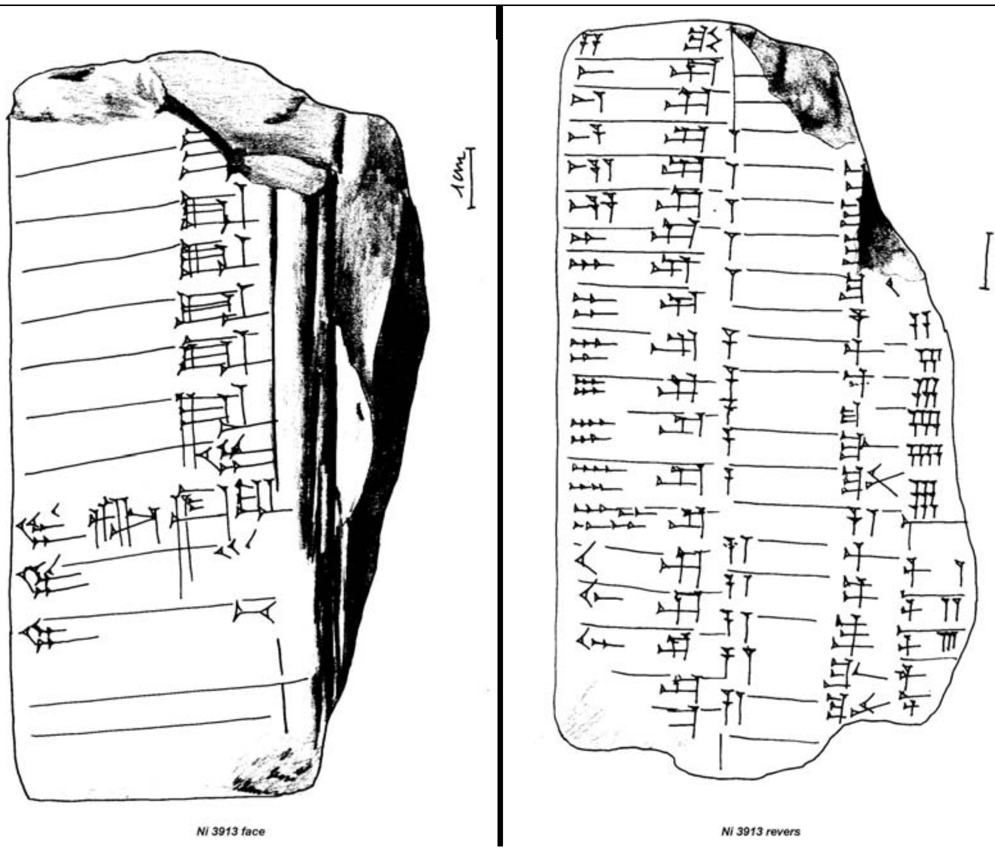
Type III tablets are small rectangular tablets containing a short extract, often a multiplication table, called "long tablets" (imgidda) in Sumerian (See tablet HS 201a, CDLI n° P254581, containing a multiplication table).

Type IV tablets are small square or round tablets, containing a short exercise, called "hand tablet" (imšu) in Sumerian (see tablet Ist Ni 18, CDLI n° P368708, containing an area calculation - Figures 2 and 3 below).

Type I, II and III tablets were used in the elementary level of education, and type IV tablet in a second stage. The vast majority of Nippur tablets is of type II, and was used as a sort of diary notebook.

[6] In order to erase signs impressed in wet clay, scribes simply rub them lightly with their finger. Tablets bear often fingerprints and erased signs covered by others.

[7] The term « metrological » refers to the measure systems (see the following page).



Ist Ni 3913, CDLI n°P229593. School type II tablet from Nippur, Istanbul Archaeological Museum (Proust 2007, copy pl. XXI).

The obverse contains a Sumerian lexical list, including mathematical terms regarding volume calculations. The reverse contains a list of measures of capacity. The right side of the tablet, which contained student copies, is lost. Note the characteristic appearance of the fracture, which results from the fact that the right columns have been written and erased several times, becoming thinner and forming a ledge.

Figure 1. *Type II tablet*

Type II tablets very often contain Sumerian texts on one side and mathematical texts on the other side. It has been shown that the text on the reverse had been studied and memorized before it is written on the obverse (Veldhuis 1997). A statistical study of correlations between the texts of the obverse and reverse of type II tablets allows a reconstruction of the order in which the various texts were studied, and therefore of the curriculum. These texts are very standardized lists (of signs, Sumerian words, phrases, measurements) and tables (metrological, numerical), which appear on many duplicates. The mathematical curriculum in Nippur, that is, the chronological sequence of different lists and tables that was to be learnt by students, may be summarily described by Table 1 below. The main function of theses texts was learning the metrological systems and sexagasimal place value notation (SPVN).[8]

Metrological lists	Capacity list Weight list Surface list Length list
Metrological tables	Capacity table Weight table Surface table Length table Height table

[8] Metrological systems (systems used for noting measures of capacity, weight, volume, surface, and length), were described in "metrological lists". Metrological tables provided a correspondence between measures and abstract numbers, that is, numbers written in sexagesimals place value notation. SPVN was used in mathematical texts. This notation used 59 digits (1-59), made of two kinds of signs: ones (vertical wedges 𒁹) and tens (oblique wedges 𒌋), repeated as many times as necessary. For example, 12 is noted 𒌋𒈫. The numbers are made of sequences of digits following a positional principle in base 60: each sign noted in a given place represents 60 times the same sign noted in the previous place (on its right). SVPN does not specifie the magnitude of the numbers. For example, the numbers 1, or 60, or 1/60 are noted in the same way (a vertical wedge 𒁹). Initial and final zeros are unnecessary, and indeed, they are not attested in any known cuneiform text. However, the absence of notation for median zero was a weakness of the system, which was corrected in later periods: in the mathematical and astronomical texts from the last centuries before our era, scribes used signs indicating the absence of a power of sixty in the positional numbers. In the transcriptions, digits are noted in the modern decimal system, and separated by dots. For example the numbers 44.26.40 which appears in Table 1 is a transcription of the cuneiform number 𒐏𒐉 𒎙𒐋 𒐏. For more details on place value notation, see (Proust 2008b).

Division / multiplication tables	Reciprocal table 38 multiplication tables (head numbers 50, 45, 44.26.40, 40, 36, 30, 25, 24, 22.30, 20, 18, 16.40, 16, 15, 12.30, 12, 10, 9, 8.20, 8, 7.30, 7.12, 7, 6.40, 6, 5, 4.30, 4, 3.45, 3.20, 3, 2.30, 2.24, 2, 1.40, 1.30, 1.20, 1.15) Square table
Tables of roots	Square root table Cubic root table

Table 1. *Mathematical curriculum in Nippur*

This mathematical curriculum was coordinated with the literary curriculum as follows:

Writing, Sumerian	Mathematics
Simple sign lists	
Thematic lexical lists	Metrological lists
Complex sign lists Contract models Proverbs	Metrological tables Numerical tables

Table 2. *Literary and mathematical curriculum in Nippur*

The texts written by students were extracted from very long lists and tables, totaling tens of thousands of items, highly standardized, in a fixed order. The stability of texts allows us to draw up a "composite text", that is, a text composed of all items found on various tablets, in the same order as they were written and taught. The composite text of lexical lists occupies several volumes of "Materials for the Sumerian Lexicon" (MSL), which represents since the publication of the first volume in 1937, an essential part of Sumerologists efforts to establish the Sumerian lexicon. The composite text of mathematical lists and tables is much smaller (published in Proust 2007).

This composite text has no material existence, since no actual tablet contains it entirely. But it probably represents fairly well what was memorized by the scribes[9]. Indeed, several piece of evidence show that these lists were memorized, at least partially, by the students. First, mathematical lists and tables are attested almost only on school tablets. Very few "reference texts", that is tables used by trained scribes for their professional activities, including teaching, are known[10]. The masters knew the tables by heart. Second, some characteristics of lexical lists, especially the logic that guides the sequential order of the items, clearly reflect the constraints of memorization. Many digressions in the lexical lists, where items attract themselves in a seemingly unexpected way, by association of ideas or homophony, could be explained by memorization processes (Cavigneaux 1989, Veldhuis 1997).

The numerical tables are also written in a quite unexpected order, since they follow an order different from that which we feel natural from an educational point of view. First comes the reciprocal table, then the different multiplication tables in descending order: table of 50, 45, 44.26.40, etc. (See Table 1 above). The first tables seem to be the more difficult. The explanation could be the following: analysis of school tablets shows that the first sections of the lists were copied with a frequency much larger

[9] Current digital databases permit a simultaneous representation of both composite text and real texts written in available sources. The advantages of digital media over paper to represent the lexical lists in all their dimensions have been noted by Veldhuis in his study of school texts of Nippur (1997, ch. 5). He exploited these advantages in the development of his online database (DCCLT, http://cdl.museum.upenn.edu/dcclt/).

[10] One of the rare exceptions is a mathematical prism now kept in the Louvre (AO 8865, CDLI No P254391), of unknown provenance. It is a large prism carefully written and crossed by an axial hole, probably to be easily usable. This prism is a precious object which looks very different from the drafts of students. This prism could indicate that the "composite text" was not always entirely memorized by professional scribes, who needed to consult a reference text.

than the last, because the copy always starts with the beginning of the list, but continues only rarely until the last item. We can assume that, by placing the "difficult" tables in first position, the masters made sure that they were more frequently copied and recited than the simplest, placed at the end of the series of multiplication tables[11].

What exactly did it mean, for a student to learn a list or a table? The typology of tablets helps us to answer this question. In a first step, the students learnt to write short excerpts, reproducing a model on the obverse of type II tablets, then they memorized the pronunciation, they recited the excerpt, and, in the last step, they reproduced by heart a large part of the list by writing it on the reverse of type II tablet. Learning therefore inextricably combined writing and memorization.

The lists and tables memorized in elementary education were a set of linguistic and mathematics tools that were subsequently used by the scribes throughout their entire administrative or scholarly careers; these tools include repertoires of signs, dictionaries, grammatical paradigms, systems of measurement, and numerical tables. As noted above, these tools are attested mostly in school tablets and rarely among scholarly writings.

Students' drafts provide evidence that the knowledge of masters included a vast repertoire of numerical results, generally memorized and ready to be mobilized in professional practice or teaching. Thus, school texts are not childish texts, but rather reflect the knowledge shared by a community of youth and adults passed through the mold of scribal schools. These texts are "elementary" in the sense that they constitute the basic knowledge needed to take on future scribal charges. Resources

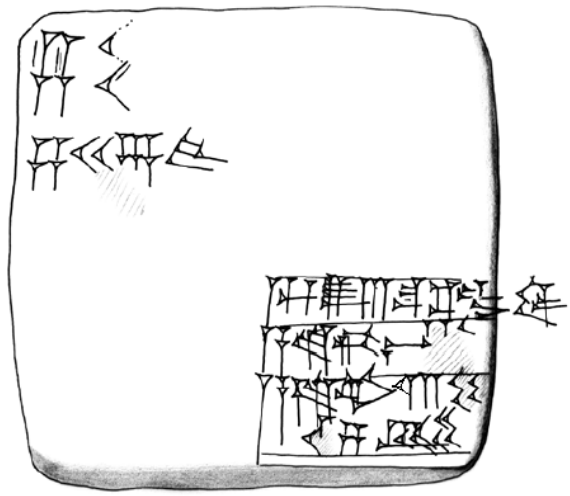
[11] This conclusion is largely based on discussions with Anne-Marie Chartier, at a workshop on education in Mesopotamia (Paris, 15/03/2006).

for elementary education were largely immaterial since they were mainly memorized by the masters.

This knowledge was not limited to the city of Nippur, but widely spread in Mesopotamia and neighboring regions. The same measure systems, the same calculation techniques, and almost the same lexical lists were taught not only in southern Mesopotamia, the cradle of the Sumero-Akkadian culture, but also to the east in Susa (west of Iran), to the north in Mari (middle valley of the Euphrates, the border between Syria and Iraq), and later to the west in Ugarit (Syrian coast). The milieu which disseminated this common knowledge was probably linked to professional scribes involved in education, who possibly circulated from one city to another. This "academic" knowledge that transcends regional boundaries could have been relatively autonomous in relation to local practices. For example, the metrology (that is, the measure system) taught in the schools of Mari and Ugarit was not the one practiced in the everyday administrative activities of these cities. Thus we see taking shape a common culture belonging to a specific segment of the population whose members, though few, were mobile and influential over a wide geographical area.

Mathematical lists and tables used in the elementary level of education form a highly structured and coherent system, whose function appears when analyzing the exercises used in a more advanced level of education. These exercises, often written on type IV tablets (see Figure 2 below), relate to multiplication, reciprocal, and calculation of area and volume. This enumeration already provides interesting information: the operations that are the subject of school training are limited to the field of multiplication. Addition and subtraction are absent. If we look more carefully at how the numbers and measures are noted in the elementary lists and tables on the one hand, and the calculation exercises on the other hand, we see that some basic principles are consistently applied. The measurements are recorded using numerical signs following an additive principle. Metrological tables provide a correspondence between, on the one hand, these measures, and on the other hand, positional numbers, noted in base 60. The exercises where multiplication and reciprocal are performed use positional numbers, with no mention of measuring units or objects counted. The multiplication and reciprocal operate only on positional number, or "abstract numbers" according to Thureau-Dangin (Thureau-Dangin 1930).

The exercises on area calculation are particularly interesting because they show how the two types of numbers act at specific and distinct stages in the calculation process. The layout of the area calculation in Ist Ni 18 (see below) shows clearly these two stages of calculation since they are written in two distinct areas of the tablet.



Ist Ni 18 : type IV school tablet from Nippur de type IV. Archaeological Museum of Istanbul. Proust 2007, pl. I

Figure 2. *Surface calculation*

School tablets reveal an original conception of numbers, where quantification and calculation fill two dissociated functions, undertaken by two different numerical systems: quantification is made by additive numerical systems (in the lower right area of the tablet Ist Ni 18), and calculation is made by a positional system (in the upper left area of the tablet Ist Ni 18). The whole set of school tablets from Nippur, including "advanced" exercises, shows the dynamic use of these tables in the practice of calculation.

Through the school tablets, the historians have access to concrete aspects of school training. The arrangement of the text on the clay tablet, the notation of numbers, and the errors provide particularly valuable clues for the reconstruction of calculation practices. But more interestingly, they show us how the masters and the erudite themselves had been trained. Our reading of advanced mathematical texts is thus transformed, since they can be tackled using mathematical tools that were inculcated into young scribes, not using our modern arithmetic and algebraic tools.

9.6 Masters' writings.

From this description of the school tablets from Nippur, a picture of a highly stereotyped education, leaving little room for pedagogical creativity, might emerge. But such an image would be the result of extrapolating too readily from information provided by extremely fragmented sources. As mentioned above, in order to reconstruct everyday life in the Old Babylonian schools, we have only the written production of beginners, as well as some literary texts. Little information has reached us concerning the parts of education that do not use writing, such as music, theater, or storytelling. Regarding mathematics, consistent evidence indicates that written artifacts represent only a part of the ancient calculation practices since mental arithmetic and concrete calculation tool played an important role (Proust 2000).

Furthermore, only the first stages of education are well known. As mentioned, we owe our sources to the recycling practices of the scribes. School tablets have been selectively preserved precisely because they had been thrown out. But what do we know about education at the more advanced levels? The available sources are less numerous and more difficult to interpret. For example, excavations of Nippur have yielded hundreds of mathematical school tablets, but only three advanced mathematical texts reached us. Even so, it is quite likely that advanced mathematics were produced at Nippur. In addition to the hazards of excavations, several different reasons may explain this paradox. Firstly, the scholarly tablets were circulating. It is possible that the excavators have unearthed tablets written in Nippur but kept in other cities, without being able to identify their origin. Indeed, the Old Babylonian mathematical tablets bear no indication of place, date or author name. Another explanation is unfortunately plausible. The mathematical tablets are most of the time of unknown origin because they were bought from dealers by European and American museums as well as by private collectors. It may be that such tablets have been found at Nippur and disappeared into the opaque net of the antiquities trade.

What is an “advanced mathematical text”? In the foregoing discussion, I used the term “advanced” in a vague sense to designate texts other than elementary school texts. But one could make more subtle distinctions and identify various types of writings, such as the production of advanced students, texts written by masters for teaching purposes, and purely erudite texts. The boundary between these types is difficult to trace, and reliable criteria are often lacking. Identifying the function of a text in relation to teaching practices is possible only on a case by case basis. Since the archaeological context is usually unknown, this analysis is usually based primarily on internal evidence. In the following, I shall provide two samples of such analysis, both related to the algorithm of reciprocal calculation. The first sample is diachronic and aims to highlight that the function of a text, including its use in education, can change over time (more details can be found in Bernard & Proust 2008). The second is synchronic and aims to show how the link between teaching and scholarship is a two-way relationship (see Proust 2010).

Reciprocal tables provide a sample of text which was a school text in certain periods, and was not in other periods. The earliest known reciprocal tables are dated from the so called Neo-Sumerian period, that is, the late third millennium B.C.. The context is the emergence and consolidation of the first centralized states, which have dominated much of Mesopotamia and neighboring regions[12]. The policies of these states were characterized by centralized control based on social and economic standardization of writing, metrology, accounting, etc., accomplished through a series of "reforms". These reforms are known mainly due to references found in some royal hymns which were used thereafter in education. For example, in the text “Shulgi B”, widely used in Old Babylonian scribal schools, the neo-Sumerian king Shulgi is supposed to have standardized metrology and to have developed the scribal schools. The interesting aspect of this story is the link established between standardization and development of schools. Indeed, schools played a major role in the creation of new standards, as well as in their wide dissemination. The reform of weights and measures, with the creation of a single and coherent system for all units of measure, results from this context. It is possible that the place value notation was invented in connection with this standardization process. Anyway, the few Neo-Sumerian reciprocal tables provide the earliest known evidence of place value notation. Furthermore, the clay tablets on which the earliest numerical tables are written are fine objects, showing a mastery of the cuneiform

[12] These states were ruled by two king dynasties in two periods separated by one century: the Akkad dynasty, 2300-2200 (Sargon and successors), and the Ur III dynasty, 2100-2000 (Ur-Nammu, Shulgi and successors).

writing which is not that of a young beginner[13]. The Neo-Sumerian reciprocal tables reflect the activity of the scholars who developed and implemented royal policies of standardization. These tables, as well as the multiplication tables that seem to have appeared later, subsequently were used for elementary education.

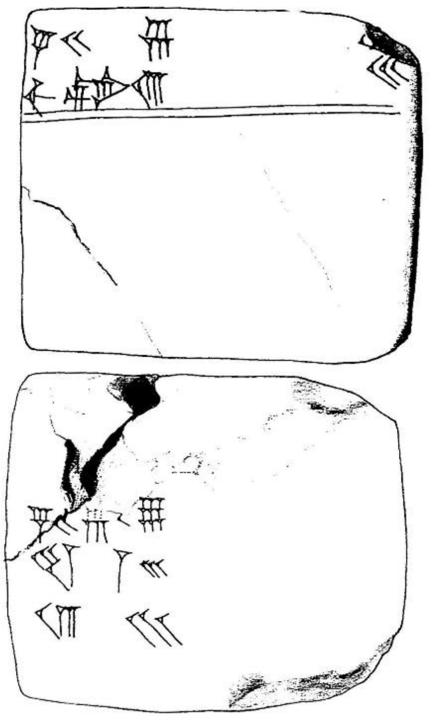
Now let us travel in time by crossing over a millennium and a half. We find again the same reciprocal tables, but in a radically different context. Mathematical tablets dated from the Hellenistic period (ca. 3rd century B.C.) were found among the remains of the great libraries of Uruk and Babylon, belonging to priests. These tables were used in teaching, but no longer in the elementary education of children learning the basics of writing and arithmetic. In this late period when cuneiform writing was disappearing, this kind of table belonged to the specialized training of young scientists already literate in Greek and Aramaic. The reciprocal tables were still school texts, but not in the same way as in the Old-Babylonian context. This example shows how the function of a text may change in history, and thus how it may be used as a resource by masters in different ways. The presence of reciprocal tables in Hellenistic libraries shows the importance that the later scholars who knew cuneiform attached to the preservation of ancient intellectual heritage.

To sum up, the tables created at the end of the third millennium in a scholarly environment linked to political power, were not necessarily primarily school texts. Subsequently, their content was widely disseminated through communities linked to scribal schools, and used in elementary education in the Old Babylonian period. Finally, reciprocal tables were incorporated into a frozen body of writing belonging to a "canonical" scholarly heritage, probably compiled in the early first millennium, and, in the Hellenistic period, these tables were transmitted in quite closed religious

[13] See Ist Ni 374, CDLI n° P257557.

circles. The function of the same table was by turn linked to engineering, elementary education, and antiquarism.

The second sample allows us to grasp another aspect of the relationship between erudite texts and school texts. This sample concerns an algorithm used in the Old Babylonian period in order to calculate reciprocals of regular numbers[14] which do not belong to standard tables. This algorithm is found in numerous school tablets, such as the following example (fig. 3)[15].



Ist Ni 10241, CDLI n° P368962. Type IV school tablet from Nippur. Istanbul Archaeological Museum. Proust 2007, pl. XLVIII

Obverse

4.26.[40]

its reciprocal 13.30

Reverse

4.26.40	9
40!	1.30
13.30	

Figure 3. *Algorithm for reciprocal calculation*

[14] A regular number in a given base (here in base 60) is a number whose reciprocal can be written with a finite number of digits. These numbers are products of divisors of the base, therefore, in base 60, their decomposition into prime factors does not include factors other than 2, 3 or 5. The ancient Mesopotamian mathematicians certainly knew all the one-digit regular numbers (given in standard tables), and probably all two-digit regular numbers, as well as a large stock of larger regular numbers (three or more digits). Their algorithms, including the division which was performed by means of multiplication by reciprocal, were mainly based on regular numbers.

[15] In translation, the exclamation point after 40 means that indeed the scribe should have written 40 on his tablet, but in fact wrote something else (in this case, he wrote 41 instead of 40).

The algorithm is based on decomposition into regular factors. The regular number for which the reciprocal is sought, here 4.26.40 (in the Mesopotamian base 60 system), is decomposed into factors belonging to known standard tables, here 6.40 and 40, and then the reciprocal of these factors, here 9 and 1.30, are multiplied the one with the other. The result is the sought reciprocal, here 13.30. Note that the factors appear in the trailing part of the number to be factorized. For example, the number 4.26.40 ends with 6.40, thus 4.26.40 is divisible by 6.40[16].

The calculation could be summarized as follows:

4.26.40 = 6.40 × 40,

thus

recip(4.26.40) = recip(6.40) × recip 40) = 9 × 1.30 = 13.30

Another text of unknown provenance, kept at the University of Philadelphia under the inventory number CBS 1215, contains many calculations of this kind. It is a large multi-column tablet (three columns on the obverse and three columns on the reverse), divided into 21 sections. The first section begins with the number 2.5, the second with the number 4.10, etc.: Each entry is double the previous. Section 8 contains the following calculation:

4.26.40	9
40	1.30
13.30	2
27	2.13.20
	4.26.40

Note that the beginning of the calculation is identical to that of the school tablet Ist Ni 10241 described above. However, after finding 13.30, the inverse of 4.26.40, the scribe continues the computation by seeking the reciprocal of 13.30, which provides of

[16] For this reason, Friberg gave the name “trailing part algorithm” to this method (Friberg 2000, p. 103-5).

course the original number 4.26.40. Each section of the large tablet CBS 1215 contains the calculation of the reciprocal of the entry (direct sequence), followed immediately by the calculation of the reciprocal of the reciprocal (inverse sequence). Almost all the known school exercises of reciprocal calculation, whatever their provenance, legally or illegally excavated, contain one of the calculations contained in the large tablet CBS 1215. So one might think that tablet CBS 1215 was a kind of textbook, and was used by masters to prepare exercises for students.

However, a closer observation of the text leads to doubt about this too simple explanation. First, the samples found in the school exercises are extracted only from the direct sequences of the large tablet CBS 1215. Secondly, the choice of regular factors in the decompositions follows fixed rules in direct sequences, but is much freer in reciprocal sequences. These observations suggest that the relationship between the large tablet CBS 1215 and the small exercises is the reverse of that which is generally supposed. In the large tablet, it would appear that the existing school material was compiled, systematized, developed and reorganized to produce a text whose objectives are not only teaching, but also searching for generalization and justification of the algorithm. Such a text would be intended to communicate certain mathematical results to peers rather than to convert knowledge into teaching materials. In fact, it is likely that the two processes, transmission and innovation, were not exclusive and that they interacted with each other in ways to which we do not clearly grasp.

The two cases briefly mentioned above show the difficulty of describing precisely what resources were available to masters (or built by them). The principal reason lies in the nature of our sources, which are fragmentary and do not always permit us to capture the complexity of the relationship between education and scholarship.

9.7 Conclusion

What can one say, finally, on the process of making resources for teaching in the Mesopotamian context? Various pieces of evidence show that the knowledge taught at an elementary level constituted a large body which was completely memorized by the experienced scribes, including masters. The knowledge of the master is thus largely embedded in their memory. But a school *tablet* does not necessarily contain a school *text* in the sense that this text has not always been specifically elaborated for the purpose of education. A text inscribed on a school tablet may belong to the cultural background of the scribes, and may have been transmitted unchanged, as is the case with dictionaries, multiplication tables or trigonometric tables today. Sometimes there is little difference between a text written by a young student and a text in some way belonging to the resources of the master. At a more advanced level, we deal with mixed processes of creating and transforming knowledge. The development of exercises is connected with the invention and explanation of new mathematical concepts. These innovations could have emerged from the school activity itself, in the context of a network between scholars. The resources of masters result therefore from a complex and two-way process between learning and scholarship, involving memory, oral communication, writing, and probably material artifacts.

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